Equilibrium

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Enrollment Numbers:
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Introduction

Mechanics is the physical science concerned with the dynamical behavior (as opposed to chemical and thermal behavior) of bodies that are acted on by mechanical disturbances. It is a fundamental science for engineering, as it helps to analyze and design various structures and machines. Mechanics has a long history, dating back to ancient times when Archimedes studied buoyancy and levers. The most comprehensive and widely used theory of mechanics is Newtonian mechanics, which is based on the laws of motion and gravity formulated by Isaac Newton in the 17th century. Newtonian mechanics can accurately describe the behavior of most objects and phenomena in engineering.

However, there are some situations where Newtonian mechanics fails, such as when the speed of a body is close to the speed of light, or when the size of a body is very small. In these cases, more advanced theories, such as Einstein’s relativity and quantum mechanics, are needed to explain the observed phenomena. But the concept of Equilibrium won’t require them

Notable Scientists in mechanics

Archimedes, who studied buoyancy and levers in the 3rd century BC

Isaac Newton, who formulated the laws of motion and gravity in the 17th century

James Joule, who experimentally found the mechanical equivalent of heat in the 19th century

Extra Byte

- Mechanical equivalent of heat: A principle that says motion and heat can be converted into each other.
- Historical significance: A key concept for understanding energy conservation and creating thermodynamics in the 1800s.

Equilibrium

An object is in Equilibrium in a reference coordinate system when all the external forces and moments acting on it are balanced. This means that the net result of all the external forces and moments acting on this object is zero.

The equilibrium condition of an object exists when Newton’s first law is valid. According to Newton’s first law, an object in equilibrium will either remain at rest or maintain its constant velocity.

There are also 3 types of equilibrium
1.Stable
2.Unstable
3. Neutral

Concept Of Free Body Diagrams

No system, natural or man-made, consists of a single body alone or is complete by itself. A single body or a part of the system can, however, be isolated from the rest by appropriately accounting for its effect. A free-body diagram (f bd) consists of a diagrammatic representation of a single body or a subsystem of bodies isolated from its surroundings but shown under the action of forces and moments due to external actions

- Book on table: The book and the table exert forces on each other and the ground. The free-body diagram of the book shows its weight and the reaction from the table.
- Car and trailer: The car pulls the trailer by a force from the engine. The free-body diagrams of the car and the trailer show the forces and moments acting on them. The net forces on the car and the trailer cause them to accelerate.

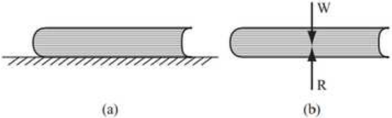


Fig. 1.8 *(a) Book on a table top (b) Free-body diagram of the book*

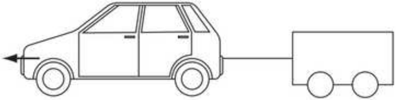


Fig. 1.9(a) *A car pulling a trailer*

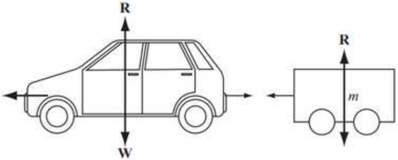


Fig. 1.9(b) *Free-body diagram of the car and trailer*

What is the idea of drawing an FBD?

- **FBD**: A diagram that shows all the forces and moments acting on a body, either in equilibrium or in acceleration. It is the basis of all mechanics analysis.
- **Example**: Two cylinders in a V-groove. The FBDs of each cylinder show their weight and the reactions from the groove and the other cylinder. The FBD of the two cylinders together shows only the external forces. The internal forces cancel out.

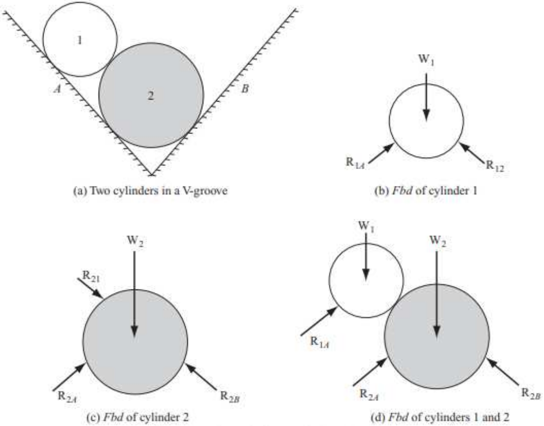
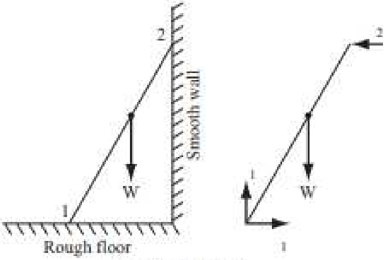
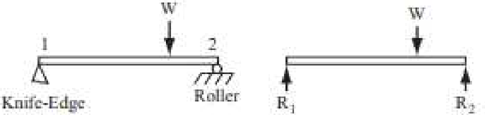


Fig. 3.2 *Two cylinders resting in a V-groove*

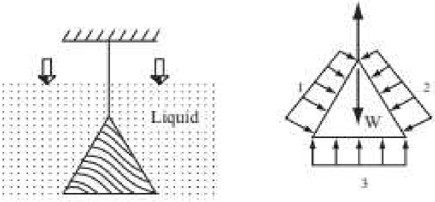
Examples Of Free Body Diagrams



(a) *Fbd* of a ladder



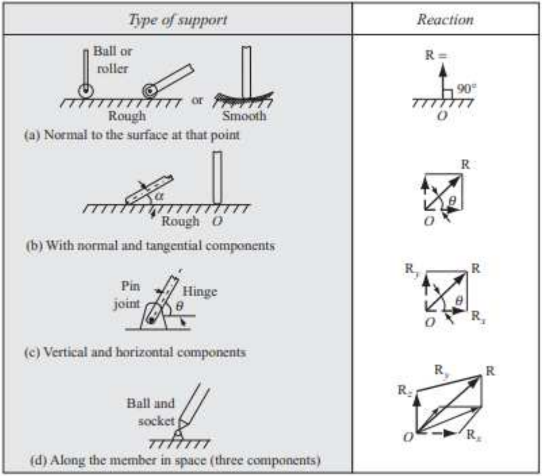
(b) *Fbd* of a beam



(c) *Fbd* of a submerged body

Reactions By Supports

Different types of supports are employed to hold structural members and components in motion. The purpose of a support is to provide a desirable reaction.



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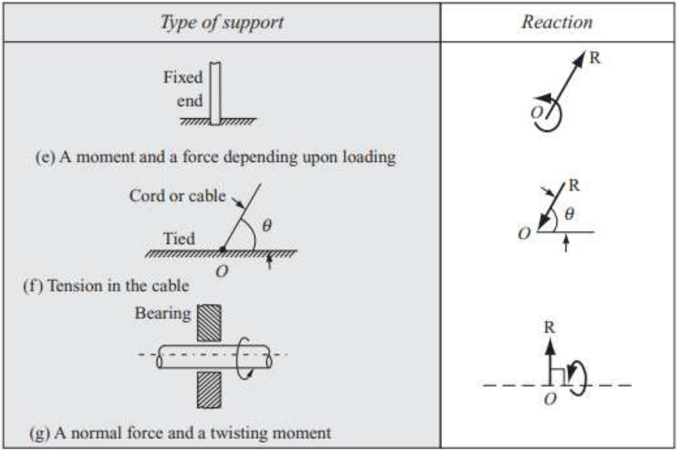


Equations of Equilibrium for a System of Concurrent Forces in a Plane

When the resultant of all the forces acting on a particle is zero the particle is said to be in equilibrium.

For the resultant R to be zero, each of the rectangular components and , must be separately equal to 0.

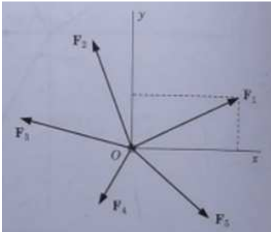


Fig. 2.17

If $R = \sqrt{Rx^2 + Ry^2} = 0$

$Rx_{net} = 0$

$Ry_{net} = 0$

Thus

$$Rx_{net} = Fx_1 + Fx_2 + Fx_3 + \ldots = 0$$

$$\sum Fx_{net} = 0$$

$$Ry_{net} = Fy_1 + Fy_2 + Fy_3 + \ldots = 0$$

$$\sum Fy_{net} = 0$$

Where

Fx_1 , Fx_2 are the components of the forces F_1 and F_2 along the X-axis.

Fy_1 , Fy_2 are the components of the forces F_1 and F_2 along the Y-axis.

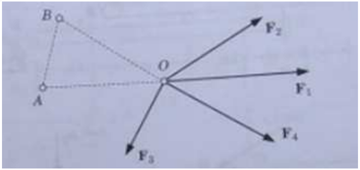
$\sum Fx_{net} = 0 \quad \ldots eq\ (1)$

$\sum Fy_{net} = 0 \quad \ldots eq\ (2)$

These are the first two Equations of Equilibrium.

Equations of Equilibrium for a System of Concurrent Forces in a Plane

Let us assume that a number of concurrent forces F_1 , F_2 , F_3 , F_4 are acting at a point O.
Let sum of moments of forces about point A be equal to zero, then



$$\sum M_{A_{net}} = 0$$

This means that -

a) Resultant of forces is 0
b) The resultant lies along line OA

Now, if we choose any other point B and suppose that sum of moment of forces acting about it is also equal to zero then,

$$\sum M_B = 0$$

Therefore, -

a) Resultant of forces is 0
b) The resultant lies along line OB

→ Since, the resultant cannot have two lines of actions (OA and OB), so the Resultant of forces is equal to zero.

$\sum M_{A_{net}} = 0$ and $\sum M_{B_{net}} = 0$ ... $eq\ (3)$

Thus, these are the Moment equations of Equilibrium.

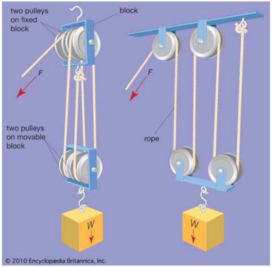
Both sets of equations are equivalent and choice depends on the problem at hand.

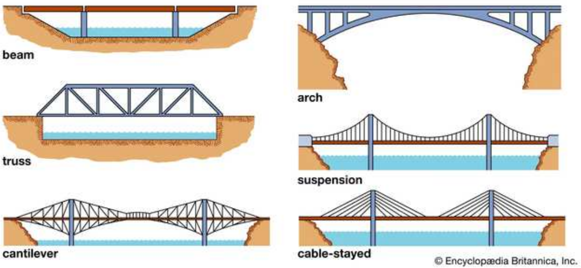
Sometimes moment equations of equilibrium can offer certain advantages by way of eliminating an unknown reaction or a force provided the moment centre is chosen to lie on the line of action of that force.

Applications to Engineering Problems

The equilibrium of forces and moments is a key concept in mechanics that helps to understand and design various systems. Some of the applications are:

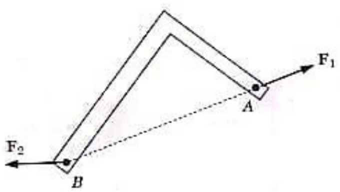
- Structural Engineering: The equilibrium equations are used to determine the stability and indeterminacy of structures like bridges and buildings.
- Mechanical Systems: The equilibrium of forces and moments are used to design systems that can resist the external forces they encounter, such as engines and machines.
- Physics and Engineering: The equilibrium of forces and moments are used to study phenomena like planetary motion and celestial mechanics.



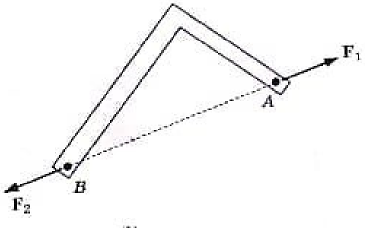


Equilibrium of Two and Three Force Members

If a rigid body is subjected to forces acting only at the two points, it is called a two force body.
Two forces can be in equilibrium only if
They are equal in magnitude
Opposite in direction
Have the same line of action.
Consider a rigid body in shape of a L-shaped plate acted upon by two forces F1 and F2 at the ends A and B



So, the forces F, and F2 cannot keep the body in equilibrium when F1 = F2.

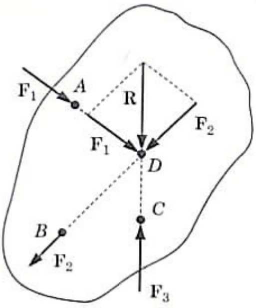


Whereas, forces, F1 and F2 can keep the body in equilibrium when F1 = F2.

Equilibrium of Two and Three Force Members

EQUILIBRIUM OF A THREE FORCE BODY

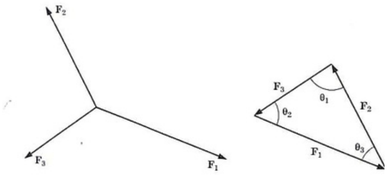
When a body is acted upon by three coplanar forces it is called a three force body.
It can be in equilibrium if:
1. The lines of action of all three forces must intersect at a single point
2. Sum of all the three forces acting on the member must be equal to zero



To prove the above statement let us consider a rigid body with three non-parallel forces F1, F2 and F3, acting at points A, B and C respectively
F1 & F2 are concurrent forces acting on a body on point D
Replace F1 and F2 by their resultant R acting at the point D.
F3 and the resultant R (of the forces F1 and F2) can keep the body in equilibrium if they have the same lines of action or are collinear.
Hence, the three forces are concurrent

Equilibrium of Two and Three Force Members

.The triangle law for these forces can be stated as belowform
Three concurrent forces in equilibrium must form a closed triangle of force when drawn in head to tail fashion as shown



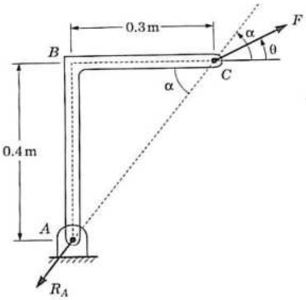
Consider the triangle of force shown. Law of sines (also called Lami's theorem) can be used

$$\frac{F_1}{\sin\theta_1} = \frac{F_2}{\sin\theta_2} = \frac{F_3}{\sin\theta_3}$$

If a body is in equilibrium under the action of three non-parallel forces, above method often simplifies the solution rather than using the equilibrium equations.

Numericals

Question) A body ABC of negligible weight is hinged at A with a force F acting at its end C. Determine the angle 0 which this force should make with the horizontal to keep the edge AB of the body vertical.



Solution: The body ABC is a *two force* body.

At the point C force F is acting at angle 0 with the horizontal and at the point A the reaction R of the hinge is acting in an unknown direction.

These two forces should be ***equal, opposite and collinear*** for the body to be in equilibrium.

To be collinear both forces should act along the line joining points A and C such that the edge AB remains vertical.

Therefore,

0

0 = α

0.4

tan α =

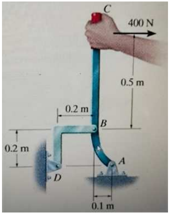
= 3.333

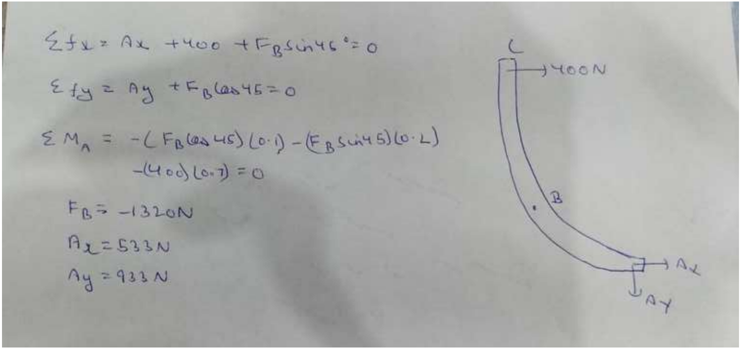
0.3

α = 53.13° Ans.

Numericals

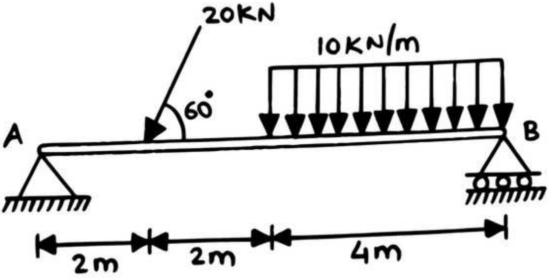
Draw a FBD for the member CBA. Determine the reaction forces at pins A and B.





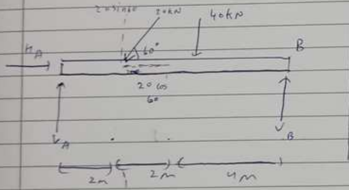
Numericals

Find the reactions at supports A and B for the beam shown in figure below:



Numericals

Sol =) To find reaction forces at A and B, make FBD of given system



$\Sigma F_x = 0$
$\Sigma F_y = 0$
$\Sigma M = 0$

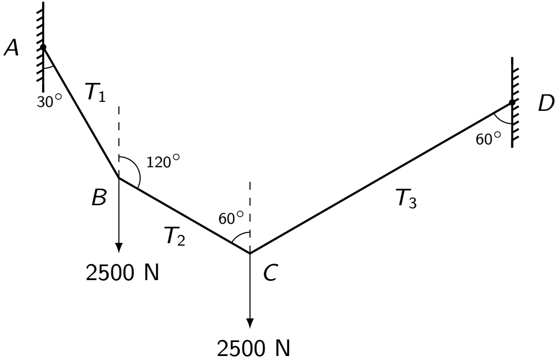
Rectangular load
$= 10 kN \times 4m$
$= 40 kN$

$\Rightarrow \Sigma F_x = 0$	$\Sigma M_A = 0$	$\Sigma F_y = 0$
$\Rightarrow H_A - 20\cos 60 = 0$	$20\sin 60 \times 2 + 40 \times 6 - R_B \times 8 = 0$	$V_A + V_B - 40 - 20\sin 60 = 0$
$H_A = 20\cos 60$	$\Rightarrow 274.641 - R_B \times 8 = 0$	$\Rightarrow V_A - 23 = 0$
$H_A = 10\ kN$	$V_B = 34.33\ kN$	$V_A = 23\ kN$

Net Reaction at $A = \sqrt{H_A^2 + V_A^2} \Rightarrow 25.079\ kN$

$\theta = \tan^{-1}\left(\frac{V_A}{H_A}\right) \Rightarrow \theta = 66.501°$

Two equal loads of 2500 N are supported by a flexible string *ABCD* at points *B* and *C*. Find the tensions in the portions *AB*, *BC* and *CD* of the string.



Numericals

Applying equations of equilibrium at point B,

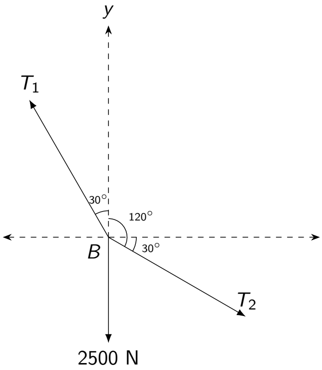
$$\boxed{\Sigma F_x = 0}$$

$$-T_1 \sin 30^\circ + T_2 \cos 30^\circ = 0$$

$$T_1 = \frac{T_2 \cos 30^\circ}{\sin 30^\circ} \tag{1}$$

$$\boxed{\Sigma F_y = 0}$$

$$T_1 \cos 30^\circ - 2500 - T_2 \sin 30^\circ = 0 \tag{2}$$



Numericals

Substituting for T_1, we get:

$$T_2\left(\frac{\cos 30^\circ}{\sin 30^\circ}\right)\cos 30^\circ - T_2 \sin 30^\circ = 2500$$

$$T_2 = 2500$$

$$T_1 = \frac{T_1 \cos 30^\circ}{\sin 30^\circ} = \frac{2500 \times 0.866}{0.5}$$

$$T_1 = 4330N$$

$$\boxed{T_1 = 4330\text{N}}\ \boxed{T_2 = 2500\text{N}}$$

Numericals

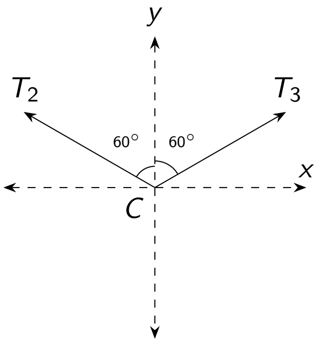
Applying equations of equilibrium at point C,

$$\boxed{\Sigma F_x = 0}$$

$$-T_2 \cos 30^\circ + T_3 \cos 30^\circ = 0$$

$$T_3 = T_2$$

$$\boxed{T_3 = 2500\text{N}}$$



Thank you for listening.