Physics Presentation (Group 9)

Derivation for mass-energy equivalence [$E = mc^2$]

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Introduction

In 1905, Albert Einstein published a paper titled *"On the Electrodynamics of Moving Bodies"*, in which he described the **Special Theory of Relativity**. Using this theory, he derived an equation showing mass-energy equivalence:

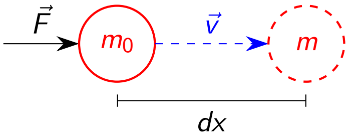
$$E = mc^2$$

This implied that mass and energy are one and the same and are interchangeable.

Let us derive this equation.

Derivation

Let us take a particle with resting mass m_0, applying force $\vec{F}$ on the particle displaces it by dx with velocity $\vec{v}$.



Calculating the work done,

$$
\begin{aligned}
dW &= \vec{F} \cdot d\vec{x} \\
&= \left(\frac{d\vec{p}}{dt}\right) \cdot d\vec{x} \\
&= v \cdot d\vec{p} \\
&= v \cdot d\left(m\vec{v}\right) \\
&= v \cdot \left[(mdv) + (vdm)\right]
\end{aligned}
$$

$$\boxed{dW = mvdv + v^2 dm} \qquad (1)$$

Derivation

To find *mvdv* using the equation for relativistic mass as derived in previous presentations:

$$m = \frac{m_0}{\sqrt{1 - \frac{v^2}{c^2}}}$$

Squaring both sides,

$$m^2 = \frac{m_0^2 \cdot c^2}{c^2 - v^2}$$

$$\boxed{m^2c^2 - m^2v^2 = m_0^2c^2} \qquad (2)$$

Differentiating equation (2):

$$c^2 2mdm - (m^2 vdv + v^2 dm) = 0$$

[Both c and m_0 are constants]

Derivation

Dividing both sides by $2m$, we get:

$$c^2dm - mvdv - v^2dm = 0$$

$$\boxed{mvdv = c^2dm - v^2dm}$$

Substituting this value in equation (1):

$$dW = (c^2dm - \cancel{v^2dm}) + \cancel{v^2dm}$$

$$\boxed{dW = c^2dm} \tag{3}$$

Assuming the initial kinetic energy of the particle is 0, let the kinetic energy after applying the force be K. Applying Work-Energy Theorem, we get:

$$dW = dK$$

Derivation

Integrating equation (3) and applying limits:

$$\int_0^K dK = \int_{m_0}^{m} c^2 dm$$

$$[K - 0] = c^2[m - m_0]$$

$$\boxed{K = (m - m_0)c^2} \tag{4}$$

Equation (4) gives the relation for relativistic kinetic energy.

$$K = mc^2 - m_0c^2$$

Here, rest mass energy is m_0c^2, which is also the potential energy of the particle.

Derivation

We know,

$$\text{Total Energy} = \text{Kinetic Energy} + \text{Potential Energy}$$
$$E = (mc^2 - \cancel{m_0 c^2}) + \cancel{m_0 c^2}$$

$$\boxed{E = mc^2}$$

Hence proved.

Significance

The resulting equation:

- Provides a universal relationship between mass and energy.
- Removes the distinction between mass and energy and shows that they are interchangeable.
- Has been experimentally confirmed with nuclear reactions like fission and fusion.
- Is also experimentally verified by pair production of electron-positron and also their annihilation.

End

Thank you for listening.